

# Comparison of Torsional Constants and Shear Areas of Beams

## 1. Introduction

This document presents a comparison of the torsional constants and shear stiffness factors (or shear areas) of various cross-sectional beams. The results are obtained from FEMAP, NE/Nastran and MSC.Nastran. The torsional constants are compared with results calculated from finite element models composed of solids, while the shear stiffness factors are compared with data published by Guttman and Wagner (Reference 2).

## 2. Model Definition and Background

### 2.1 Torsional Constant

For any beam cross-section, the torsional constant is:

$$J = \frac{TL}{\phi G}$$

where:      J = torsional constant  
              T = torque  
              L = length of the beam  
               $\phi$  = angle of twist  
              G = shear modulus of elasticity (modulus of rigidity)

The theoretical torsional constants are only available for simple geometries, such as rod and rectangular cross-sections. One rod beam model and one rectangular beam model are created as test cases. To find the torsional constant, a torque is applied on these finite element models composed of solid elements. When the angle of twist is obtained, torsional constants are calculated using the above equation.

The beams' cross-sections and dimensions for these test cases are included in Figure 1, Table 1 and Table 2 shown below. All dimensions are in inches.

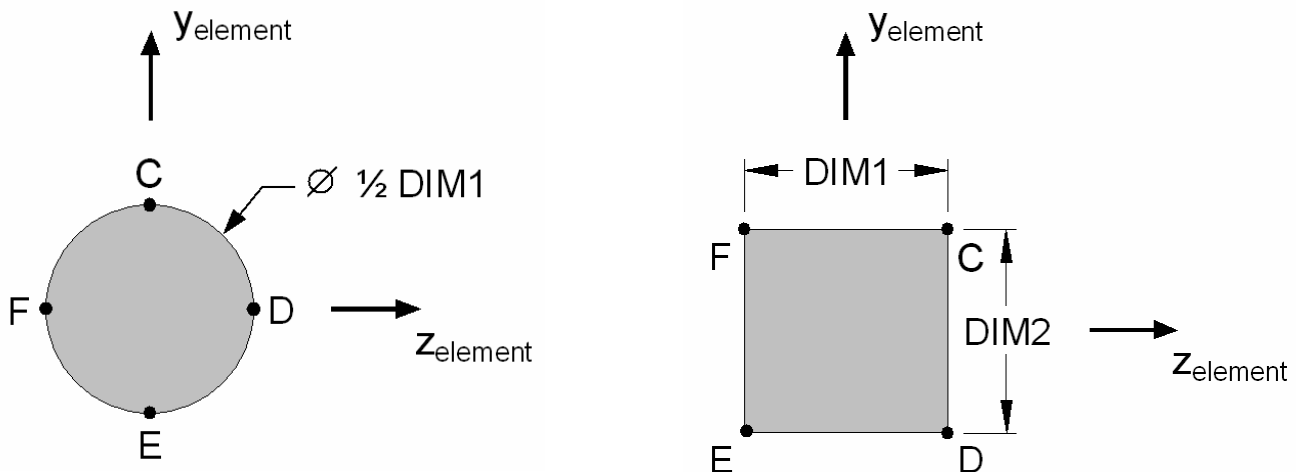
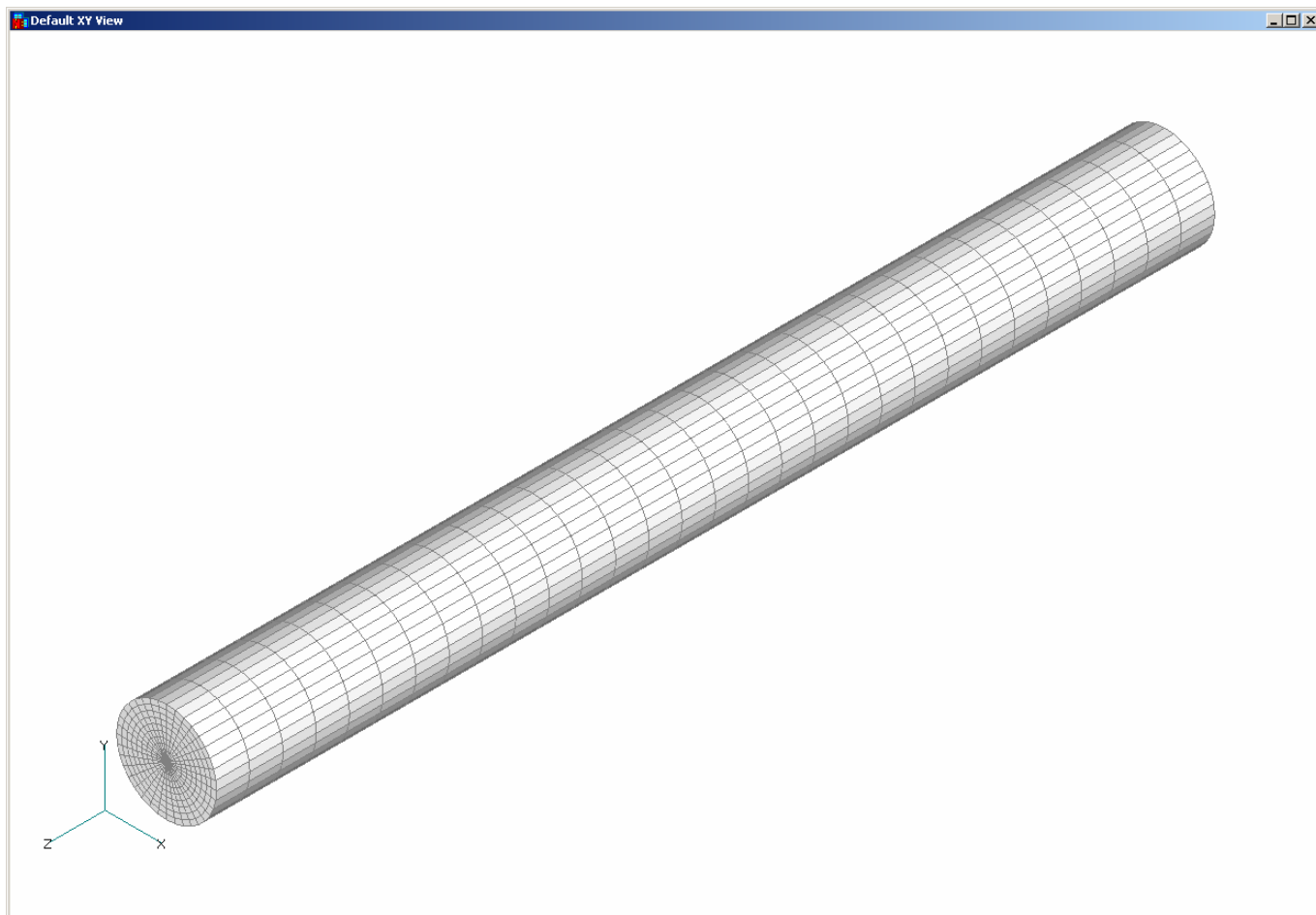


Figure 1. Cross-Sections of ROD Beam and BAR Beam

**Table 1. Dimensions of ROD Beam**

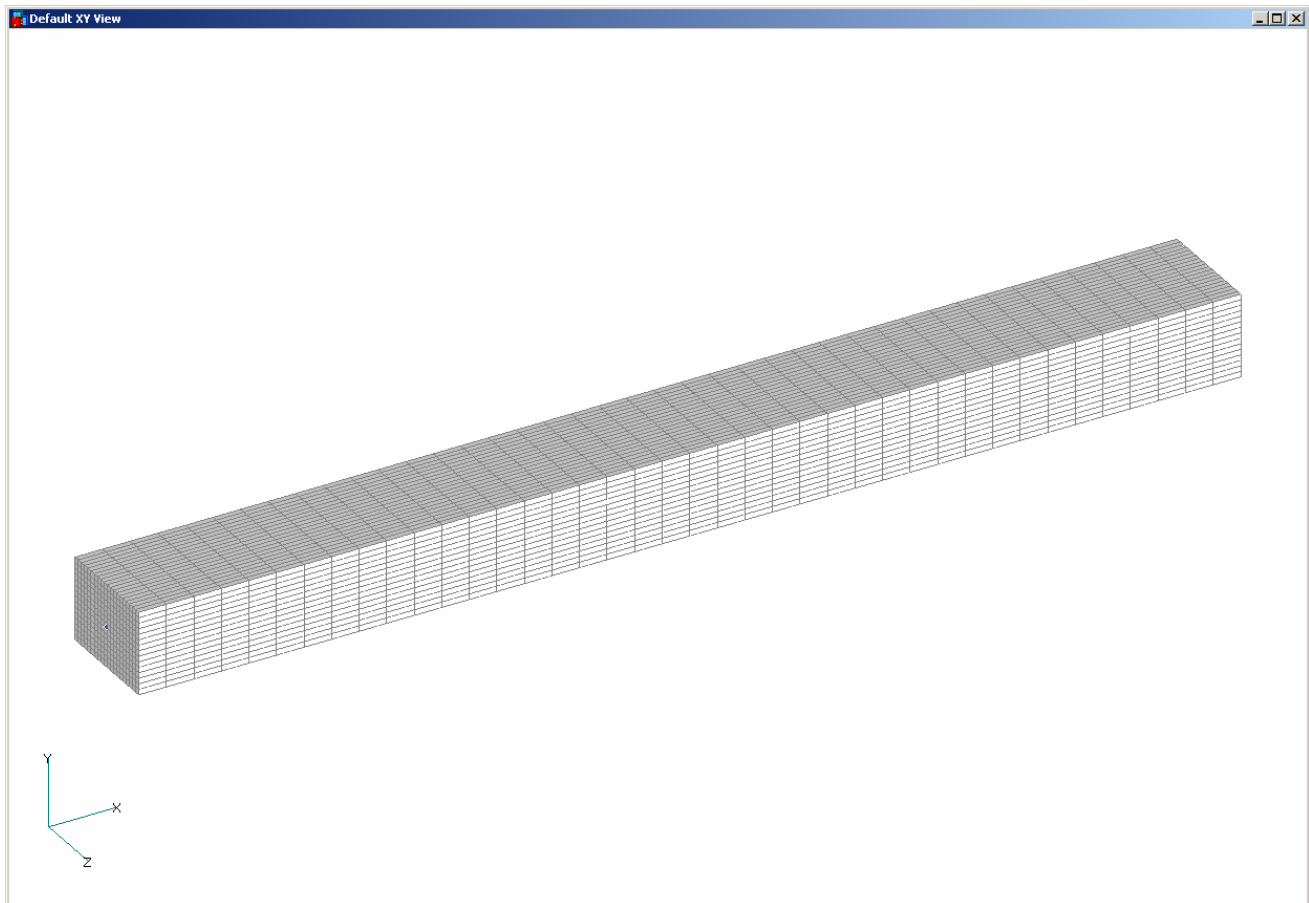
	Dimension (in)
D1 (radius)	1.50
Length	30.00



**Figure 2. FEA Model of ROD Beam**

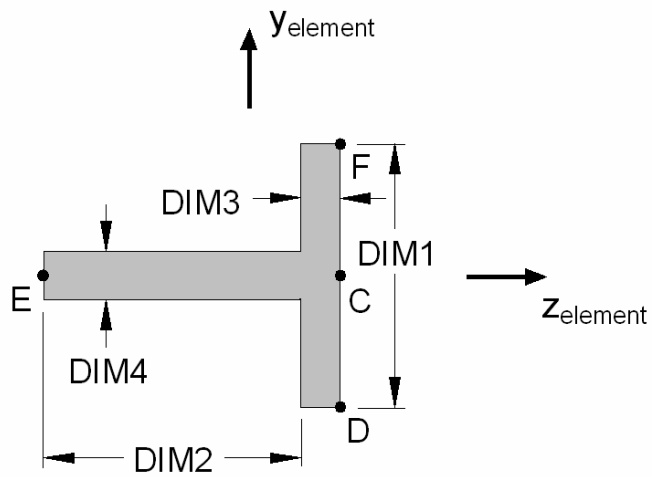
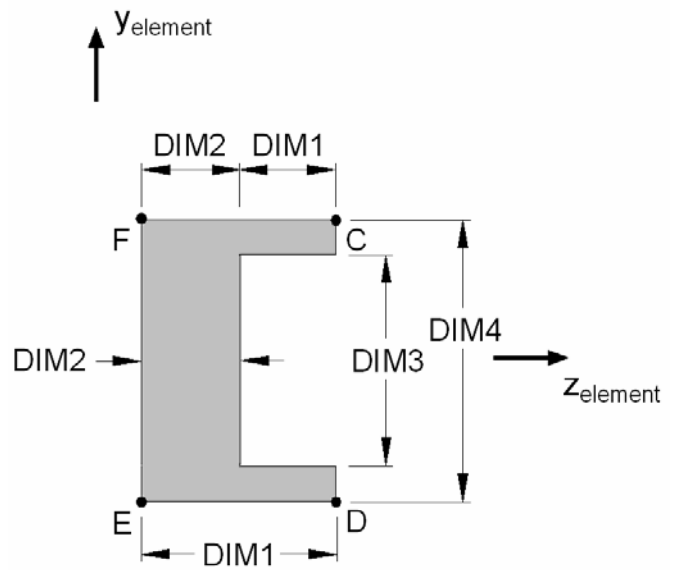
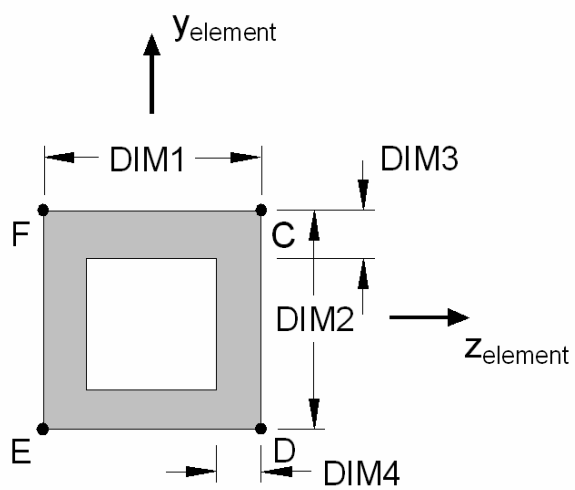
**Table 2. Dimensions of BAR Beam**

	Dimension (in)
DIM1	2.00
DIM2	1.50
Length	20.00



**Figure 3. FEA Model of BAR Beam**

Three more models, a box beam, chan1 beam, and T1 beam are considered, and their torsional constants are calculated and compared using the same approaches. These cross-sections are shown in Figure 2.

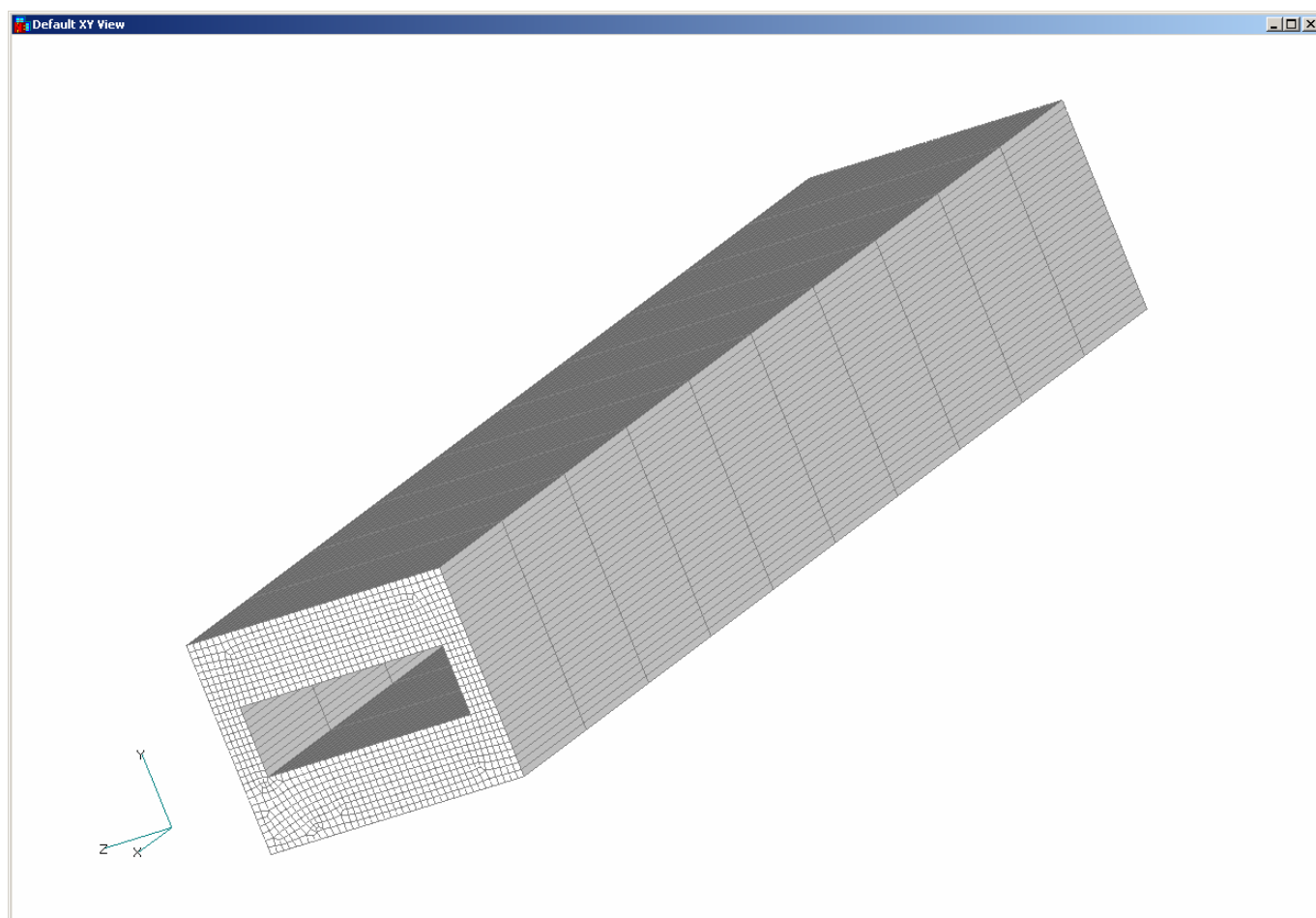


**Figure 4. Cross-Sections of BOX Beam, CHAN1 Beam, and T1 Beam**

Their dimensions are given in Tables 3, 4 and 5 below. All dimensions are in inches.

**Table 3. Dimensions of BOX Beam**

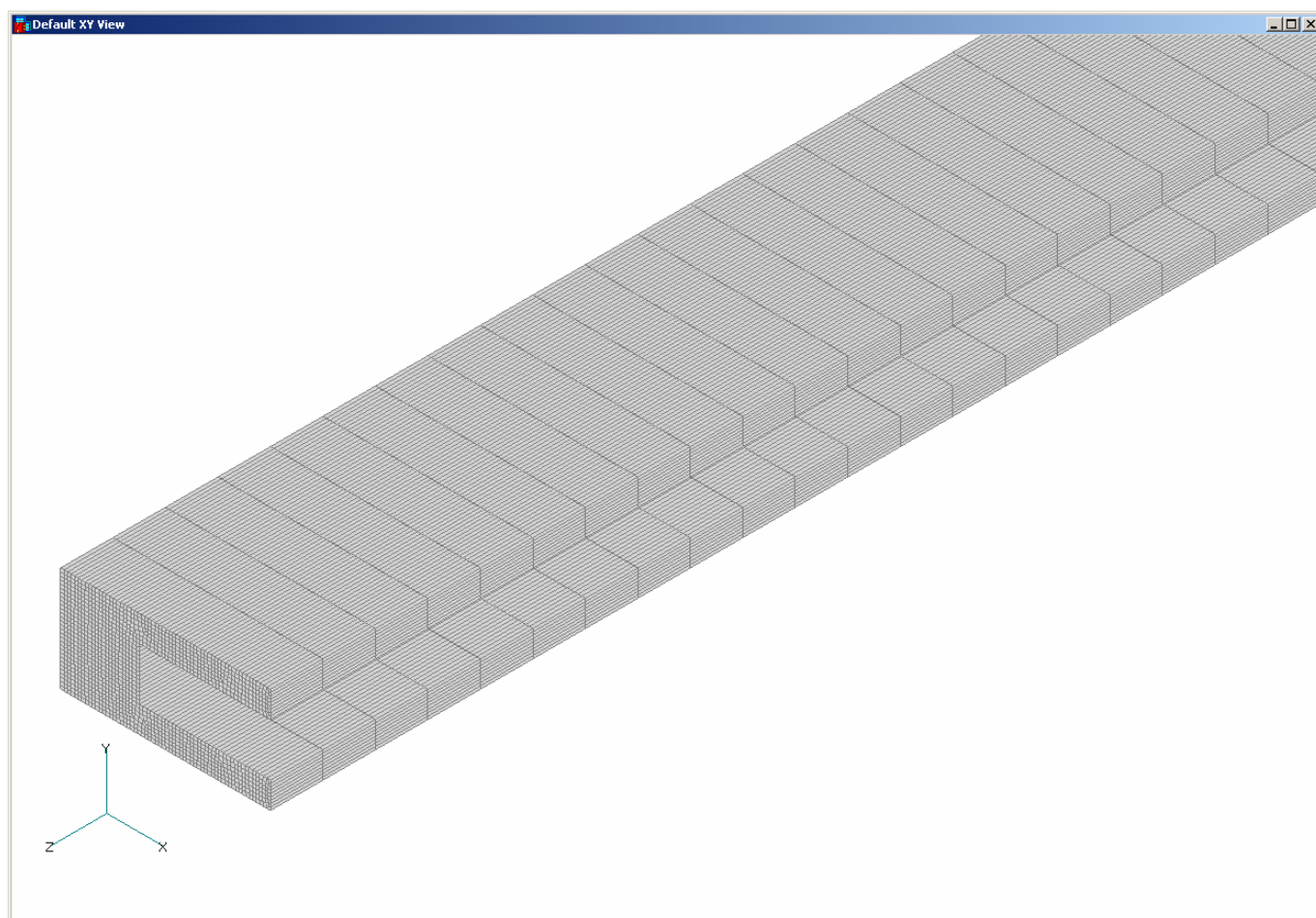
	Dimension (in)
DIM1	2.00
DIM2	1.50
DIM3	0.50
DIM4	0.20
Length	10.00



**Figure 5. FEA Model of BOX Beam**

**Table 4. Dimensions of CHAN1 Beam**

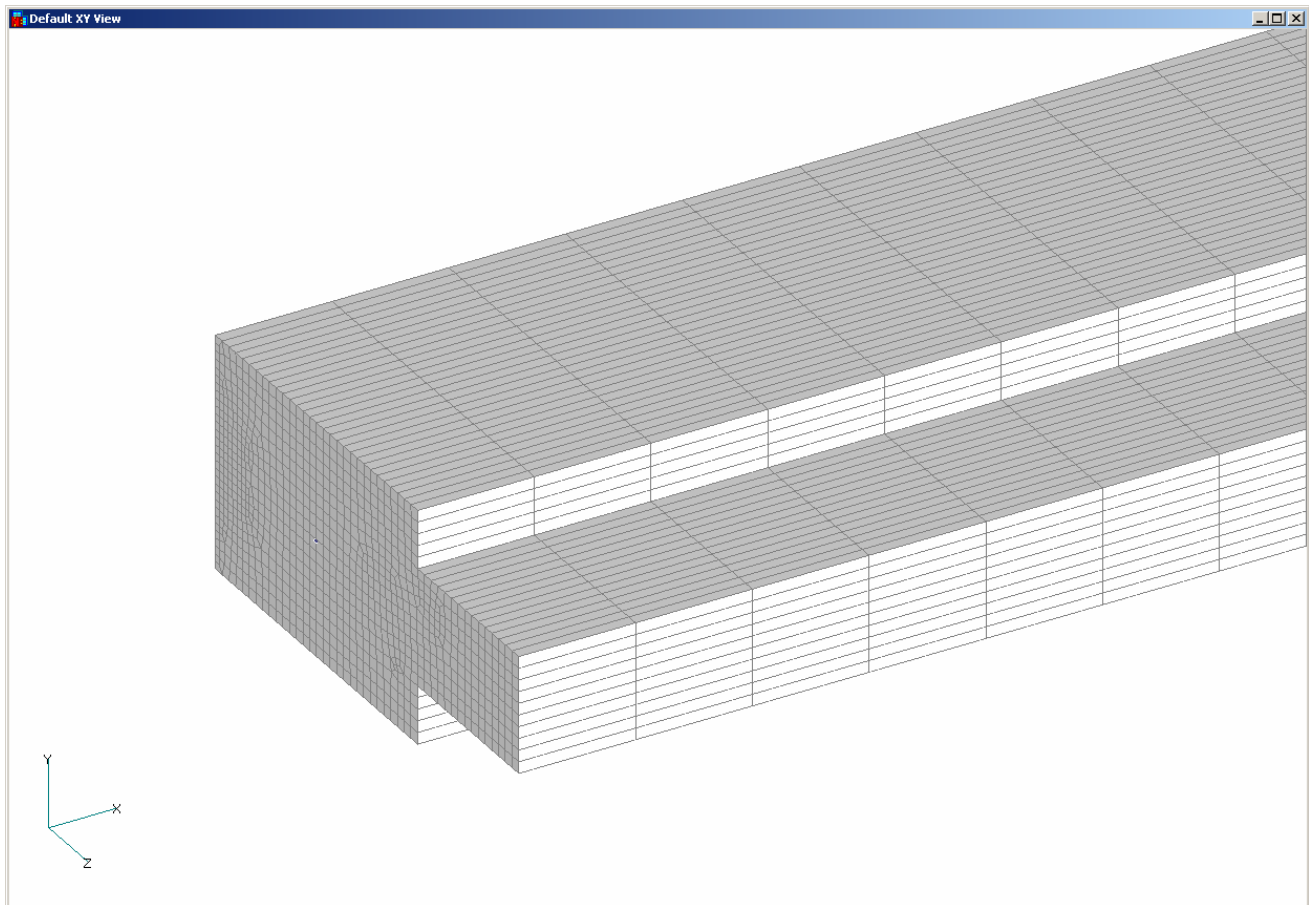
	Dimension (in)
DIM1	1.25
DIM2	0.75
DIM3	0.50
DIM4	1.00
Length	40.00



**Figure 6. FEA Model of CHAN1 Beam**

**Table 5. Dimensions of T1 Beam**

	Dimension (in)
DIM1	1.00
DIM2	0.75
DIM3	1.50
DIM4	0.50
Length	45.00



**Figure 7. FEA Model of T1 Beam**

## 2.2 Shear Stiffness Factor and Shear Area

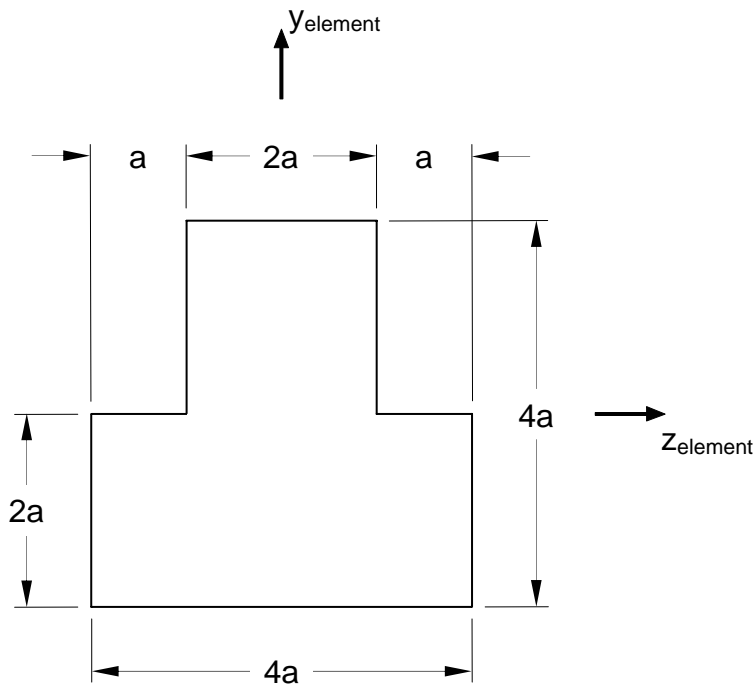
For any beam cross-section, the shear area is:

$$\text{Shear Area} = K * \text{Area}$$

where:  $K$  = shear stiffness factor

The comparison is based on the data given in Reference 2, in which the shear stiffness factor for a thick T-beam is calculated. The T-beam cross-section is shown in Figure 8, where  $a = 1.0\text{in}$  and  $\text{area} = 12\text{in}^2$ . NE/Nastran and MSC.Nastran output shear stiffness factor directly, while FEMAP outputs shear area.

FEMAP's shear area is calculated using the above equation.



**Figure 8. T-Beam Cross-Section**

### 3. Results Comparison

#### 3.1 Torsional Constant

All models created to determine the torsional constant use a Young Modulus  $E = 2.85E+7\text{psi}$  and Poisson's Ratio  $\nu = 0.485$ . Equal and opposite torques of magnitude 100 are applied at the ends.

**Table 6. Results ROD Beam and BAR Beam**

Beam Type	Torsional Constant		
	Theory*	FEA Model	Error (%)
Rod	7.90	7.95	0.68
Bar	1.23	1.21	1.17

\*NOTE: The theoretical value is taken from Reference 1.

**Table 7. Results BOX Beam**

	Torsional Constants	Error (%)
Solid FEA Model	1.00	
FEMAP	0.98	2.12
NE/Nastran	0.97	2.31
MSC.Nastran	0.75	24.48



**Table 8. Results CHAN1 Beam**

	<b>Torsional Constants</b>	<b>Error (%)</b>
Solid FEA Model	0.10	
FEMAP	0.09	4.45
NE/Nastran	0.09	4.33
MSC.Nastran	0.12	-26.19

**Table 9. Results T1 Beam**

	<b>Torsional Constants</b>	<b>Error (%)</b>
Solid FEA Model	0.42	
FEMAP	0.35	15.12
NE/Nastran	0.35	14.63
MSC.Nastran	1.16	-178.34

The torsional constants from FEMAP and NE/Nastran have less error in both the closed cross-section (BOX) and opened cross-section (CHAN1 and T1).

### 3.2 Shear Stiffness Factor

The shear areas calculated by commercial software, such as FEMAP, NE/Nastran, ANSYS and MSC.Nastran, are independent of Poisson's ratio. Similar to Reference 3, we pick Poisson's ratio ( $\nu$ ) equal to zero for comparison.

**Table 5. Shear Stiffness Factor of Thick T-beam**

$\nu = 0$	<b>Reference 1</b>	<b>FEMAP</b>	<b>NE/Nastran</b>	<b>MSC.Nastran</b>
y/a	0.677	0.679	0.679	0.333
z/a	0.740	0.742	0.742	0.667

**Table 6. Error (%) of Shear Stiffness Factor Compared to Reference 1**

$\nu = 0$	<b>FEMAP</b>	<b>NE/Nastran</b>	<b>MSC.Nastran</b>
y/a	-0.295%	-0.295%	50.812%
z/a	-0.270%	-0.270%	9.865%

FEMAP and NE/Nastran have less than 1% error in both Y and Z-shear stiffness factors. MSC.Nastran has poor results especially for Y-shear factor.

#### 4. References

1. Pilkey, Walter D., *Analysis and Design of Elastic Beams: Computational Methods*, Wiley, 2002.
2. Guttman, F. and Wagner, W., *Shear Correction Factors in Timoshenko's Beam Theory for Arbitrary Shaped Cross-sections*, Computational Mechanics, Vol. 27, 2001, pp. 199-207.
3. Yu, Wenbin, Volovoi, Vitali V., and Hodges, Dewey H., *Validation of the Variational Asymptotic Beam Sectional (VABS) Analysis*, Proceedings of the 42<sup>nd</sup> Structures, Structural Dynamics and Materials Conference, Seattle, Washington, April 16-19, 2001, AIAA Paper 2001-1530.