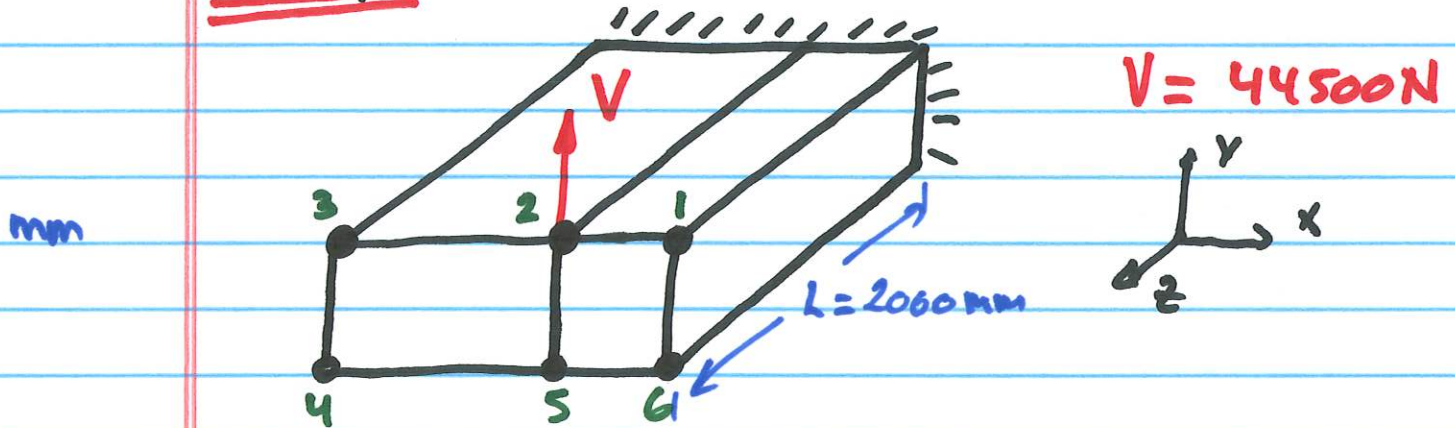


Example:



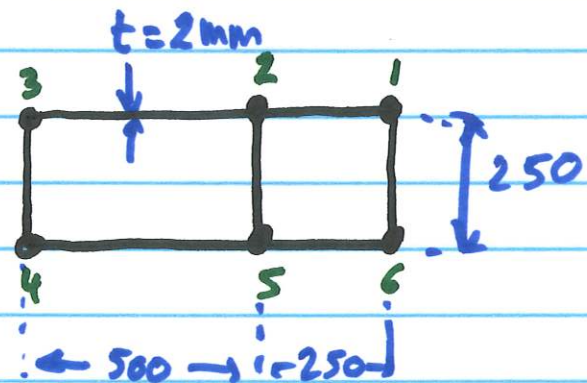
Determine the vertical deflection at point 2.

$$E = 69000 \text{ N/mm}^2$$

$$G = 25900 \text{ N/mm}^2$$

$$B_1 = B_3 = B_4 = B_6 = 650 \text{ mm}^2$$

$$B_2 = B_5 = 1300 \text{ mm}^2$$



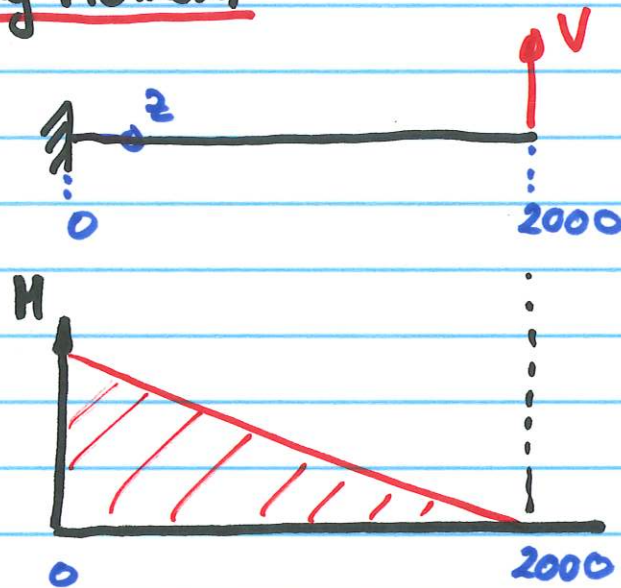
The deformation at point is given by Castigliano's theorem.

$$q_2 = \int_0^L \frac{M}{EI} \frac{\partial M}{\partial V} dz + \int_0^L \oint \frac{q}{Gt} \frac{\partial q}{\partial V} ds dz$$

Bending due to moment M

Bending due to shear (Torsion)

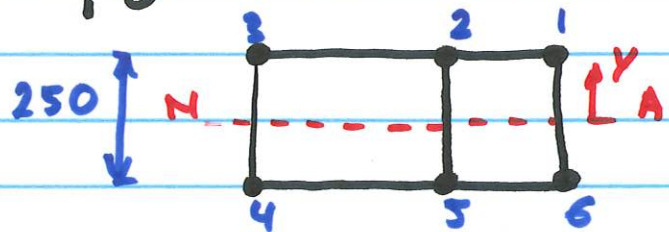
Bending Moment



$$M = V(z - 2000)$$

- Assuming that panels/skin/webs are non-effective in bending (CASE 1)

$$I_{xx} = \int y^2 dA = \sum_i y_i^2 B_i$$



$$I_{xx} = 4(650 \text{ mm}^2)(\pm 125 \text{ mm})^2 + 2(1300 \text{ mm}^2)(\pm 125 \text{ mm})^2$$

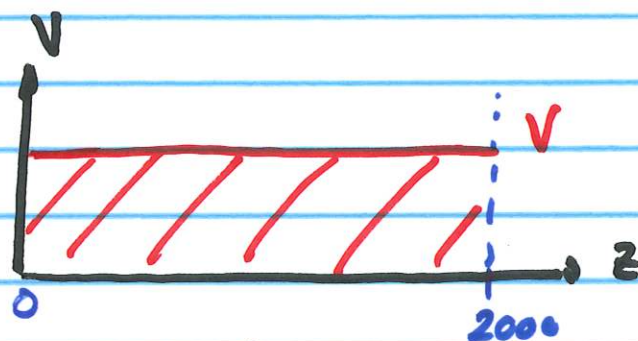
$$\Rightarrow I_{xx} = 81.25 \cdot 10^6 \text{ mm}^4$$

The deformation due to bending can now be determined.

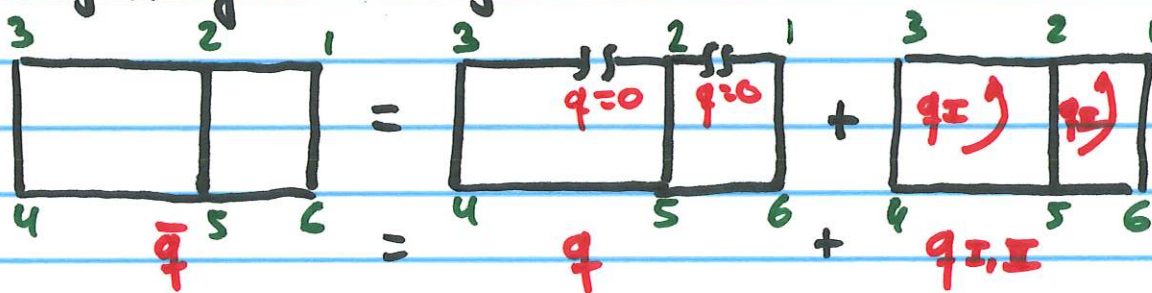
$$\begin{aligned}
 q_2|_{\text{Bend}} &= \int_0^L \frac{M}{EI} \frac{\partial M}{\partial V} dz \quad \begin{cases} M = V(z-2000) \\ I = 21.25 \times 10^6 \\ E = 69000 \end{cases} \\
 &= \int_0^L \frac{V(z-2000)}{EI} \cdot (z-2000) dz \\
 \Rightarrow q_2|_{\text{Bend}} &= \frac{V}{EI} \int_0^{2000} (z-2000)^2 dz
 \end{aligned}$$

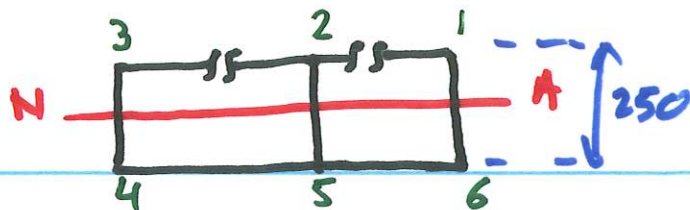
$$q_2|_{\text{Bend}} = 21.17 \text{ mm}$$

Shear Diagram



The shear flows are constant throughout the length of the wing-box.



Cell I

$$q_{23} = 0$$

$$q_{34} = q_{23} - \frac{V \cdot B_3 \cdot y_3}{I_{xx}}$$

$$= 0 - \frac{44500 (125) (650)}{81.25 \cdot 10^6} \Rightarrow \boxed{q_{34} = -44.5}$$

$$q_{45} = q_{34} - \frac{V \cdot B_4 \cdot y_4}{I_{xx}} = 0$$

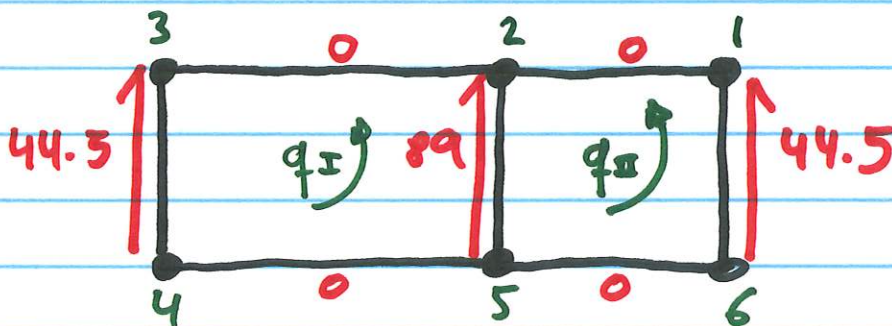
Common Web

$$q_{25} = (\cancel{q_{12}} + \cancel{q_{23}}) - \frac{V \cdot B_2 \cdot y_2}{I_{xx}} \Rightarrow \boxed{q_{25} = -89}$$

Cell II

$$q_{56} = q_{45} + q_{25} - \frac{V}{I_{xx}} B_5 y_4 \Rightarrow \boxed{q_{56} = 0}$$

$$q_{61} = q_{56} - \frac{V}{I_{xx}} B_6 y_6 \Rightarrow \boxed{q_{61} = 44.5}$$



Determine $q_I / q_{II} \Rightarrow$ Two eqts.

$$\left(\frac{d\theta}{dz} \right)_I = \left(\frac{d\theta}{dz} \right)_{II} ; \frac{d\theta}{dz} = \frac{1}{2AG} \oint \frac{q}{t} ds$$

$$\bullet \left(\frac{d\theta}{dz} \right)_I = \frac{1}{2A_I G t} \left[q_I (500 + 250 + 500 + 250) \dots \right. \\ \left. - 44.5 (250) + 89 (250) \dots \right. \\ \left. - q_{II} (250) \right]$$

$$\Rightarrow \left(\frac{d\theta}{dz} \right)_I = \frac{1}{2A_I G t} [1500 q_I - 250 q_{II} + 11125]$$

$$\bullet \left(\frac{d\theta}{dz} \right)_{II} = \frac{1}{2A_{II} G t} \left[q_{II} (\overset{250}{500} + \overset{250}{250} + \overset{250}{500} + \overset{250}{250}) \dots \right. \\ \left. + 44.5 (250) \dots \right. \\ \left. - 89 (250) \dots \right. \\ \left. - q_I (250) \right]$$

$$\Rightarrow \left(\frac{d\theta}{dz} \right)_{II} = \frac{1}{2A_{II} G t} [-250 q_I + 1000 q_{II} - 11125]$$

$$A_I = 2A_{II}$$

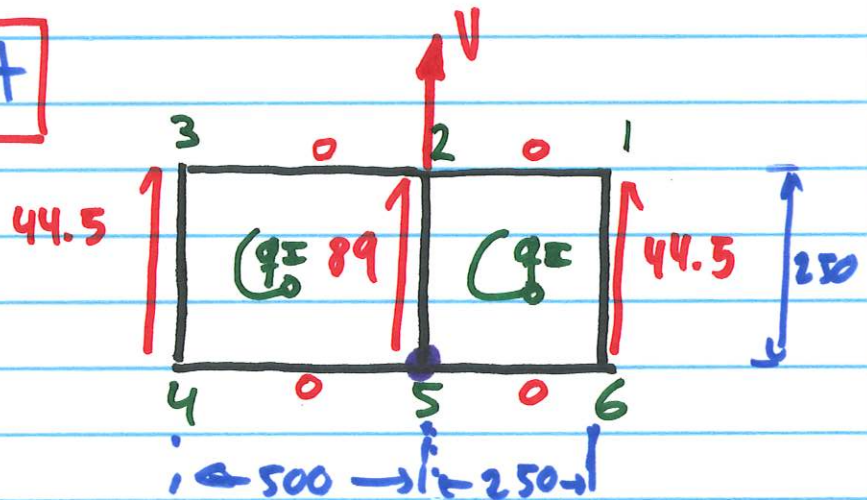
$$\left(\frac{d\phi}{dz}\right)_I = \left(\frac{d\phi}{dz}\right)_{II}$$

$$\frac{1}{2A_I k} [1500q_I - 250q_{II} + 11125] = \frac{1}{2A_{II} k} [-250q_I + 1000q_{II} - 11125]$$

$$\Rightarrow 750q_I - 125q_{II} + \frac{11125}{2} = -250q_I + 1000q_{II} - 11125$$

$$\Rightarrow 1000q_I - 1125q_{II} + 1.6688 \cdot 10^4 = 0 \quad \text{--- eqt 1}$$

$$\Sigma M_{ext} = \Sigma M_{int}$$



$$\curvearrowright^+ (\Sigma M_{ext})_5 = 0$$

$$\curvearrowright^+ (\Sigma M_{int})_5 = q_I \overset{(500)}{\underset{(250)}{\nearrow}} (250) - 44.5 \overset{(250)}{\underset{(250)}{\nearrow}} (500) + q_I \overset{(250)}{\underset{(250)}{\nearrow}} (500) \dots$$

$$+ q_{II} \overset{(250)}{\underset{(250)}{\nearrow}} (250) + 44.5 \overset{(250)}{\underset{(250)}{\nearrow}} (250) + q_{II} \overset{(250)}{\underset{(250)}{\nearrow}} (250)$$

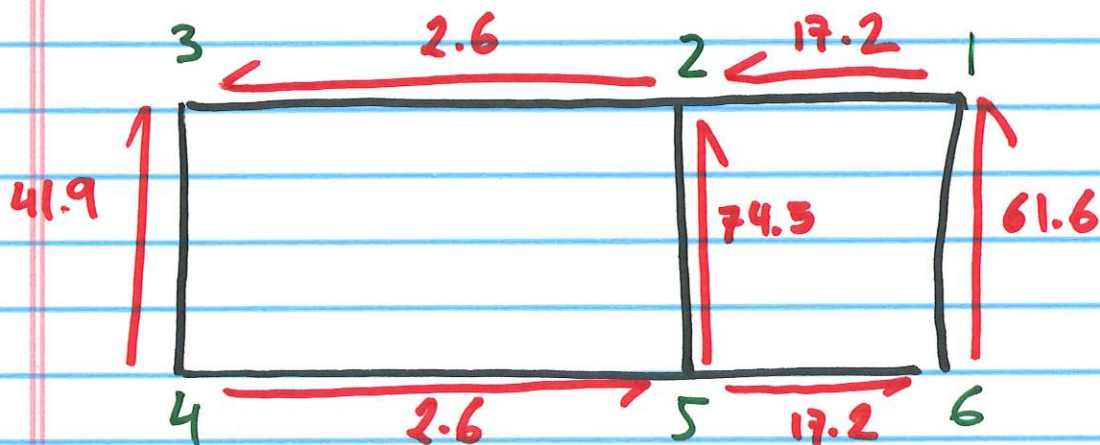
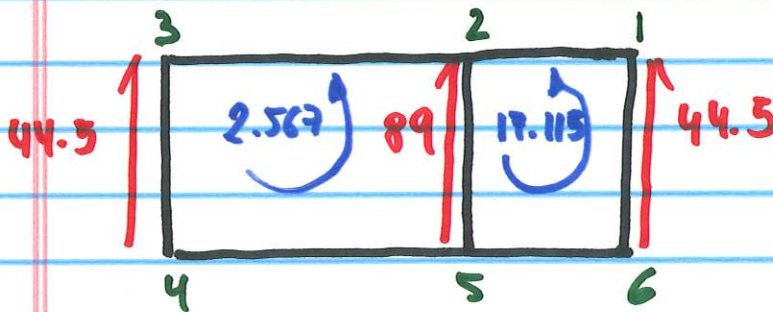
$$\Rightarrow -250,000 q_I - 125,000 q_I + 278,1250 = 0 \quad \text{--- eq 2}$$

Solving $\begin{cases} \text{eq 1} = 0 \\ \text{eq 2} = 0 \end{cases}$ for $q_I \neq q_{II}$

$$q_I = 2.567 \text{ N/mm}$$

$$q_{II} = 17.115 \text{ N/mm}$$

The final shear distribution is



The deformation due to shear (Torsion)

$$q_2|_{sh} = \int_0^L \oint \frac{q}{Gt} \frac{\partial q}{\partial V} ds \cdot dz$$

$$\begin{cases} q = \frac{-V}{I_{xx}} B_i y_i \\ \frac{\partial q}{\partial V} = -\frac{1}{I_{xx}} B_i y_i \end{cases}$$

Substituting

$$q_2|_{sh} = \int_0^L \oint \overbrace{\left(\frac{-V B_i y_i}{I_{xx} G t} \right)}^q \overbrace{\left(-\frac{1}{I_{xx}} B_i y_i \right)}^{q/V} dz$$

OR

$$q_2|_{sh} = \frac{1}{G \cdot V \cdot t} \int_0^L q \frac{\partial q}{\partial V} ds \cdot dz$$

$$= \frac{1}{G \cdot V \cdot t} \int_0^L q^2 ds \cdot dz$$

$$\Rightarrow q_2|_{sh} = \frac{1}{G \cdot V \cdot t} \int_0^L [\bar{q}_{12}^2 s_{12} + \bar{q}_{23}^2 s_{23} + \dots] dz$$

$$\Rightarrow \boxed{q_2|_{sh} = 2.54 \text{ mm}}$$

Finally, the total deformation

$$q_2 = q_{2|end} + q_{2|sh}$$
$$= 21.17 + 2.54$$

$$\Rightarrow q_2 = 23.7 \text{ mm}$$

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